



REMARKS ON EXPERIMENTS AT NAL

S. D. Drell
Stanford University

July 1968

In this seminar my remarks will be confined to two classes of experiments that meet the following two conditions:

1. They probe new ranges of physical parameters that can be reached only by the 200-BeV accelerator.
2. They place special demands on the experimental facilities because of very small cross sections or extreme angular restrictions.

We will talk about:

- (i) Two-body reactions at high s and t values. Determination of their structure at the extreme regions of s and t is very interesting and probably very important and requires measurements of very small cross sections.
- (ii) Scattering of neutral and charged hadrons from electrons in the target atoms. With the Weston accelerator one will in this way learn about their vector matrix elements in reactions such as

$$K_L + e \rightarrow K_S + e$$

$$\pi^\pm + e \rightarrow \pi^\pm + e$$

$$\Lambda^0 + e \rightarrow \Lambda^0 + e$$

which have very high counting rates but from which the scattered particles recoil into very narrow angular regions.

(a) Two-body reactions at high s and t .

Experiments at laboratory momenta up to 30 BeV/c corresponding to $s \equiv 2M_N^2 + 2M_N E_{\text{lab}} \approx 60 \text{ BeV}^2$ and out to invariant momentum transfers of $-t \equiv (p-p')^2 \approx 25 (\text{BeV}/c)^2$ show that the elastic cross section falls to $\approx 10^{-12}$ of its peak forward value. There are theoretical conjectures, but we do not know what to expect as s and t increase further. However, if we are to study two-body processes such as

$$\pi p \rightarrow \pi N$$

$$Kp \rightarrow KN$$

$$\bar{N}p \rightarrow \bar{N}N,$$

where π , K , and p denote pion, kaon, and proton respectively, and N a nucleon (n or p) out to ranges of s and t for which

$$\frac{d\sigma(s, t)}{dt} \leq 10^{-12} \left[\frac{d\sigma(s, t=0)}{dt} \right],$$

it will be necessary to prepare and provide intense secondary particle beams of π , K , \bar{N} and perhaps hyperons that are indeed intense but not out of the question (to a theorist!). Thus to achieve a counting rate of one event/hour a flux F will be required such that

$$\left[\frac{d\sigma(s, t)}{dt} \right] (\Delta t) (\text{No. target particles}) F \geq 1 \text{ count/hour.} \quad (1)$$

We take

$$\begin{aligned}\frac{d\sigma(s, t)}{dt} &= 10^{-12} \left[\frac{d\sigma(s, t=0)}{dt} \right] = \frac{10^{-12} \sigma_{\text{tot}}^2(s)}{16\pi} \\ &= \frac{10^{-12} (30 \text{ mb})^2}{16\pi},\end{aligned}$$

$\Delta t \approx \pi (\text{BeV}^2)$ corresponding to a finite momentum transfer bite; π was chosen as a compromise between a readily acceptable larger bite to determine elastic scattering and the limitations by detector geometry if the full 2π azimuthal range of scattering angles is not covered.

$$\text{No. target particles} \approx 10^{24}.$$

This gives in Eq. (1) $F \sim 10^9$ particles/sec. Since two-body reactions are 4c events when both final particles are detected it may not be necessary to restrict the incident projectile energy within narrow momentum bounds in order to define an elastic event and therefore the above value for F for secondary particles in a broad momentum band from $\approx 10^{13}$ incident protons is not unreasonable. The inelastic background of strictly forward-produced π^0 's will limit the acceptable bin of incident momenta to define an elastic event. Of course one will want to pursue pp elastic scattering with full beam intensity to the highest s and t values possible but my remarks here are a theorist's appeal to plan to map out as much as possible of the large s and t regions as conceivably possible for all two-body processes.

What might be observed? Since I have the floor I will expose my own prejudices (theory?) developed together with Henry Abarbanel and Fred Gilman. It is important to note that we are extreme optimists in the sense that we predict no further drop in pp cross sections, for fixed t as s increases, and therefore no decrease in counting rates. The

letter published in Physical Review Letters* develops our theoretical reasons for predicting the approach to and existence of an energy independent asymptote for large t as s increases beyond 60 BeV² or $E_{\text{lab}} > 30 \text{ BeV}$.

Since that work appeared we have developed a more complete theory of this conjectured behavior, i. e. ,

$$\lim_{s \rightarrow \infty} \frac{d\sigma(s, t)}{dt} \propto G^4(t), \quad (2)$$

starting with an input expression for the interaction forces or "driving terms" and deducing there from an approximately unitary S-matrix and scattering cross section. Our goal in this work is to answer the following basic question: In electron-proton scattering one measures the matrix element of a conserved vector current for momentum transfer t between initial and final single physical nucleon states and summarizes the observations in terms of form factors. In contrast, in p-p scattering we have proposed a model containing both a strong interaction via currents, as well as a strong diffraction term summarizing inelastic contributions

* Physical Review Letters, Volume 20, No. 6, February 5, 1968, pp1280-283.

via unitarity to elastic scattering. From out of this stew of strong interactions distorting the two proton wave functions via multiple vector and Regge type exchanges, how does purée of electromagnetic form factor emerge? More directly stated, if we construct a T-matrix starting with interacting currents as the "driving term" or input contact force and then add to this the inelastic or diffraction amplitudes, what is the t dependence of the resulting scattering amplitude fully unitarized? Does it still show a $G^4(t)$ variation in the differential cross section for large t ?

There are several additional questions that can also be addressed. For example, what is going on at small t values? The forward scattering amplitude naively obtained in our letter by extrapolating the contact term from large t has approximately equal real and imaginary parts, in contradiction with experiments that fix the ratio of real to imaginary parts of the forward amplitude to be much less than one, even at present energies. Although our original model was imagined to be applicable only when $-t \gg M_N^2$, can our present approach remove this restriction and show how the observed behavior near $t = 0$ emerges? In this connection there is the very striking observation, emphasized by Feynman, that the close proportionality of $d\sigma/dt$ and $G^4(t)$ remains valid all the way to very small t . Can we also shed light on this behavior? What about the famous "breaks" in $d\sigma/dt$? Finally, once we extend our theory to $t = 0$, what can we say about the total cross section and the status of the

Pomeranchuk theorem? In particular, what is the resulting asymptotic limit of the contact interaction for $s \rightarrow \infty$?

We have now addressed these questions. We postulate that there is an elementary local interaction between two protons of the current-current form which operates in addition to the usual strong interaction dynamics leading to diffractive contributions which are customarily summarized in a Regge parametrization. We can then introduce a precise form for this current-current interaction that embodies the Wu-Yang idea; namely, our input is just a product of single nucleon matrix elements whose structure is that of the electromagnetic current. It is introduced as an additional "force": an inhomogeneous term in the fixed t dispersion relation in the energy s for p - p scattering. To this we add the usual forces leading to diffractive behavior. Next we construct an approximately unitary scattering amplitude following the procedures developed by Blankenbecler and Goldberger¹ and Baker and Blankenbecler.² They introduce the Fourier-Bessel transform of the scattering amplitude, for in the high energy regime this leads to an exceedingly simple unitarity relation from which the elastic amplitude can be recovered by a judicious mixture of quadratures and computers.

The resulting theory differs in two essential ways from related studies of the connection of p - p data and electromagnetic form factors.

(1) We have introduced a local current-current interaction in addition to the diffraction scattering one would normally contemplate. In the

models based on Yang's work,^{3, 4, 5} the form of the diffraction term itself is identified with the electric charge density. More precisely, if one writes the partial amplitude at energy s and impact parameter b as $e^{2i\delta(b, s)}$, the scattering phase $\delta(b, s)$ is interpreted in Refs. 3 and 4 as a path integral proportional to the overlap of the electric charge distributions of the colliding hadrons. (2) The S-matrix, as approximately unitarized in our approach with the Fourier-Bessel transform, also has desired analyticity properties--in particular, a unitarity cut. Our final results closely correlate with the form of Eq. (2) and hence with experimental data at energies ≈ 30 BeV over the full range of measured t values extending over many decades for $d\sigma/dt$.

As a final comment on this topic, I would recommend planning experiments to search for possible parity violating contributions in the extreme regions of large s and t . For such small cross sections (perhaps 10^{-10} of the forward peak for pp scattering), who knows if a "handedness" for the amplitude might not emerge giving rise to a polarization component in the scattering plane of form $\langle \underline{\sigma} \cdot \underline{k} \rangle$ reflecting the existence of a sense of rotation in the individual microscopic space-time cells near the light cone that are probed at high s and t ?

(b) Scattering of neutral and charged hadrons from target electrons.

Poirier⁶ has discussed the feasibility of measuring the electromagnetic form factor of the charged pion by scattering from atomic electrons as the target material in a beam of high-momentum pions. I will devote

the rest of this seminar to emphasizing theoretical interest in such measurements--for charged kaons as well as pions and for neutral beams as well.

The relevant kinematics are as follows. For an incident projectile of mass M and lab energy E_{lab} the maximum invariant square of the momentum transfer to the target electron is

$$t_{\text{max}} = \frac{-2m_e E_{\text{lab}}}{1 + M^2/2m_e E_{\text{lab}}}, \quad (3)$$

and for 90° scattering in the collision center of mass the transfer is

$$t_{90^\circ} = \frac{1}{2} t_{\text{max}}.$$

Typical numbers for an incident π beam of 100 BeV/c momentum are

$$t_{\text{max}} \approx -(290 \text{ MeV})^2,$$

$$t_{90^\circ} = (210 \text{ MeV})^2.$$

The energy and angle in the lab for the pion scattered 90° in the center of mass are roughly 60 BeV and 2 millirad respectively and correspondingly for the recoil electron, 40 BeV and 3 millirad.

The cross sections are large, exceeding 10^{-31} cm^2 for events with $|t| > \sim 1/3 |t_{\text{max}}|$ and the contribution of the pion structure is also

considerable. The mean square radius expansion of a form factor is usually written

$$F_{\pi}(t) \approx 1 - \frac{1}{6} |t| < R_{\pi}^2 > ,$$

and thus the correction to the point electrodynamic cross section is

$$\begin{aligned} \left(\frac{F_{\pi}(t)}{F_{\pi}(0)} \right)^2 &\approx 1 - \frac{1}{3} |t| < R_{\pi}^2 > \\ &\approx 1 - \frac{|t|}{(550 \text{ MeV})^2} , \end{aligned} \tag{4}$$

if we take for R_{π}^2 the ρ -dominant prediction of $< R_{\pi}^2 > = 6/m_{\rho}^2$.

For incident K^{\pm} mesons the momentum transfers are lower due to the higher K mass and Eq. (3) leads to

$$\begin{aligned} t_{\text{max}} &= - (170 \text{ MeV})^2 , \\ t_{90^{\circ}} &= - (120 \text{ MeV})^2 , \end{aligned}$$

for 100-BeV incident momentum. However in Eq. (4) these still correspond to sizable effects and the kinematical identification of the event is still quite clear. Thus for 90° center-of-mass scattering the scattered K emerges with ≈ 83 BeV at a lab angle of 1 millirad and the electron recoils with ≈ 17 BeV at 5 millirad.

On the theoretical side there is interest in an accurate determination of the pion charge radius and in particular of the difference between it and the measured nucleon radii. As shown in a recent paper,⁷ the nucleon

isovector (Pauli) radius is significantly larger than both the pion charge radius and the predictions of ρ dominance. This is due to important and known threshold contributions to the absorptive parts in a dispersion analysis. In the words of the uncertainty principle the size of the pion current distributions about the nucleon extends out as far as $\Delta x \sim c\Delta t \sim c\hbar/\mu c^2 \sim \hbar/\mu c$. For the π meson structure, however, the selection rule of conservation of G parity dictates that $\pi \rightarrow \pi + \rho$ - i.e., the range of the pion current about a pion is restricted because of the requirement to make the ρ meson rest mass in the intermediate state and $\Delta x \sim c\hbar/M_\rho c^2 \sim \hbar/M_\rho c$. This suggests that the ρ dominant prediction of a radius $6/M_\rho^2 = 0.4 f^2$ should be a better approximation for the pion size than for the nucleon. A quantitative measurement of $\langle R_\pi^2 \rangle$ and of its difference from $\langle R_{2V}^2 \rangle = 0.7 f^2$ is eagerly anticipated. To avoid theoretical uncertainties and complications in the interpretation of $e\pi$ production and of $\pi^\pm - \alpha$ scattering results, it will be necessary to do elastic scattering of pions from target atomic electrons at the momentum transfers of $> 180 \text{ MeV}/c$ first available at Serpukhov so that $1/3 |t| < R_\pi^2 > \geq 10\%$. This point is emphasized by the recent analysis of their $e\pi$ production experiments by Akerlof et al.⁸ and Mistretta et al.⁹ The latitude in theoretical formulae permits their data to be interpreted as indicating a pion radius of either $\sqrt{0.4 f^2}$ or $\sqrt{0.7 f^2}$. It is to be hoped that colliding beam experiments will in due time also contribute to our understanding of pion and kaon electromagnetic structure via the processes

$e^+ + e^- \rightarrow \pi^+ + \pi^-$ and $e^+ + e^- \rightarrow K^+ + K^-$ which map out the form factor in the time like region. Their magnitude near their respective thresholds of $4m_\pi^2$ and $4m_K^2$ will also be sensitive to the electromagnetic radii.

One can readily extend these remarks to neutral meson beams, where it is not just the corrections to point Coulomb scattering due to structure but the entire effect that is of interest, and to baryon beams with magnetic moments. Consider for example the process

$$K_L + e \rightarrow K_S + e,$$

where the electromagnetic vertex changes the CP quantum number of the K^0 meson. If the coupling at the $(K_L K_S \gamma)$ vertex is defined in terms of a transition radius by

$$-ie (p_L + p_S)^\mu \left(\frac{1}{6} q^2 < R_{tr}^2 > \right),$$

where

$$q^2 \equiv (p_i - p_f)^2,$$

then

$$\sigma \approx \frac{\pi \alpha^2}{18} < R_{tr}^2 > \frac{(s - m_K^2)^2}{s},$$

where $s = m_K^2 + 2m_e \omega_{lab}$ where m_K and m_e are the K and e masses respectively and ω_{lab} is the initial kaon energy in the lab. Thus for $\omega_{lab} \approx 100 \text{ GeV}$ and $< R_{tr}^2 > \sim (\frac{1}{2} f)^2$, $\sigma \approx 4 \times 10^{-33} \text{ cm}^2$ so there is no

counting rate problem providing triggering on the knock-on high-energy electron plus a two-body kinematics constraint successfully removes background (viz. from π^0 decay Dalitz pairs). For reference we quote the exact differential cross section

$$\frac{d\sigma}{d|t|} = 4\pi\alpha^2 \left\{ \frac{F^2(t)}{t^2} \right\} \left\{ 1 + \frac{st}{(s - m_K^2)^2} \right\},$$

in terms of the charge form factor $F(t)$ ($\rightarrow 1/6t < R_{tr}^2 >$ as $t \rightarrow 0$) and of the invariant momentum transfer

$$t = \frac{-4m_e^2 E_{lab}^2}{2m_e E_{lab} + m_K^2 + E_{lab}^2 \theta_{lab}^2} \quad (-t \gg m_e^2),$$

where E_{lab} and θ_{lab} are the incident K energy and the K scattering angle in the laboratory reference frame ($E_{lab}/m_K \gg 1$). The maximum momentum transfer is

$$t_{max} = \frac{-4m_e^2 E_{lab}^2}{2m_e E_{lab} + m_K^2} = \frac{-\left(s - m_K^2\right)^2}{s},$$

for $\theta_{lab} = 0$ corresponding to backscattering or $\theta = \pi$ in the barycentric system.

For an incident charged baryon the considerations are very much the same as above with the additional matrix elements appearing due to the spin being negligible. This is easy to understand because the heavy baryon is nonrelativistic in the barycentric system and so simple charge scattering dominates. However, for a neutral baryon ($\Lambda_0 + e \rightarrow \Lambda_0 + e$ for example) there is a significant new feature and that is the fact that the scattering measures the magnetic moment. Thus to a very good approximation we have

$$d\sigma = \frac{\pi\alpha^2}{M^2} \kappa^2 \left(\frac{dt}{t} \right) = \frac{\pi\alpha^2}{M^2} \kappa^2 \left(\frac{dk'}{k'} \right),$$

where κ is the magnetic moment in units of the Bohr magneton ($e/2M$) for the baryon of mass M , and k' is the energy in the laboratory system of the (knock on) recoiling electron. Integrating from $k' \sim 5$ GeV up to the maximum [see Eq. (3)]

$$2m_e k'_{\max} = -t_{\max} = \frac{2m_e E_{\text{lab}}}{1 + m_e^2/2m_e E_{\text{lab}}} \approx 10 \text{ GeV},$$

for $E_{\text{lab}} \sim 100$ GeV and $m \sim 1$ GeV, we have

$$\bar{\sigma} \equiv \int_{5 \text{ GeV}}^{10 \text{ GeV}} d\sigma \approx 5 \kappa^2 \times 10^{-32} \text{ cm}^2,$$

which leads once again to a reasonable counting rate for $\kappa \sim 1$ providing background can be controlled. Perhaps this will be useful for measuring magnetic moments of neutral and sufficiently long lived strange baryons.

REFERENCES

- ¹R. Blankenbecler and M. L. Goldberger, Phys. Rev. 126, 766 (1962).
- ²M. Baker and R. Blankenbecler, Phys. Rev. 128, 415 (1962).
- ³T. T. Chou and C. N. Yang, in Proceedings of the Second International Conference on High Energy Physics and Nuclear Structure, Rehovoth, Israel, 1967, edited by G. Alexander (North-Holland Publishing Company, Amsterdam, The Netherlands, 1967), pp. 348-359; T. T. Chou and C. N. Yang, Phys. Rev. (in press); T. T. Chou and C. N. Yang, Phys. Rev. Letters 20, 1213 (1968).
- ⁴L. Durand and R. Lipes, Phys. Rev. Letters 20, 637 (1968).
- ⁵C. Chiu and J. Finkelstein, circulated CERN preprint, CERN-TH-892.
- ⁶J. A. Poirier, NAL Summer Study Report C. 1-68-10, 1968.
- ⁷S. D. Drell and Dennis J. Silverman, SLAC PUB 404 (submitted to Phys. Rev. Letters).
- ⁸C. W. Alkerlof et al., Phys. Rev. 163, 1482 (1967).
- ⁹C. Mistretta et al., Phys. Rev. Letters 20, 1523 (1968).